

# The History and Mathematics of the N-Localizer for Stereotactic Neurosurgery

Russell A. Brown<sup>1</sup>, James A. Nelson<sup>2</sup>

1. Software Development, Apple 2. University of Washington School of Medicine

✉ **Corresponding author:** Russell A. Brown, russ.brown@yahoo.com

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## Abstract

This paper is a supplement to "The Origin of the N-Localizer for Stereotactic Neurosurgery" [1] and "The Mathematics of the N-Localizer for Stereotactic Neurosurgery" [2]. It clarifies the early history of the N-localizer and presents further details of the mathematics of the N-localizer.

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**Categories:** Medical Physics, Radiation Oncology, Neurosurgery

**Keywords:** stereotactic neurosurgery, stereotactic radiosurgery, image guidance, computed tomography, magnetic resonance imaging, n-localizer

## Introduction

The N-localizer has become an important neurosurgical tool that has achieved widespread use in modern stereotactic neurosurgery and radiosurgery. The N-localizer produces two circles and one ellipse in tomographic, sectional images that are obtained via computed tomography or magnetic resonance imaging (Figure 1). The relative spacing between the ellipse and the two circles precisely determines the location of the tomographic section relative to the N-localizer [3-4].

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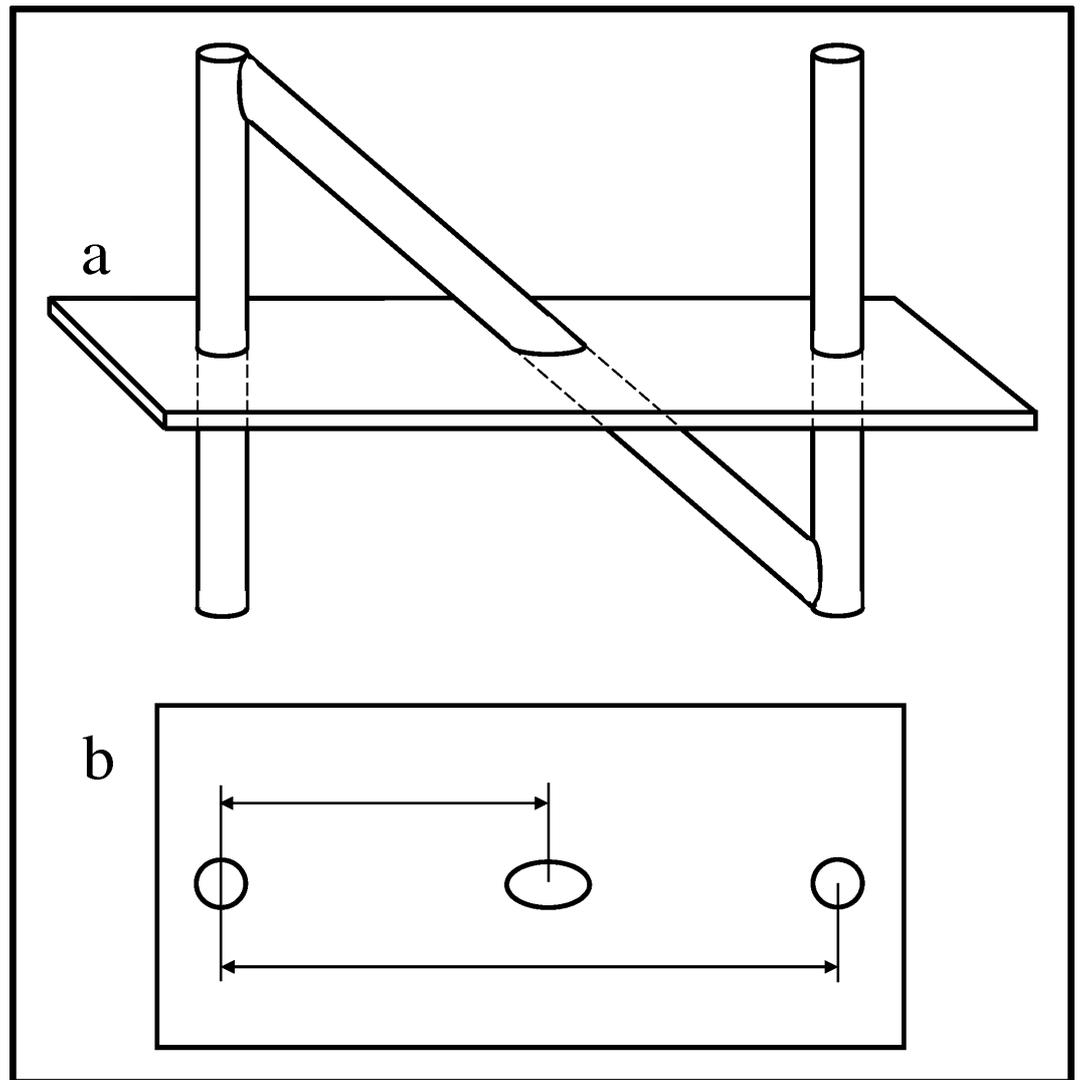
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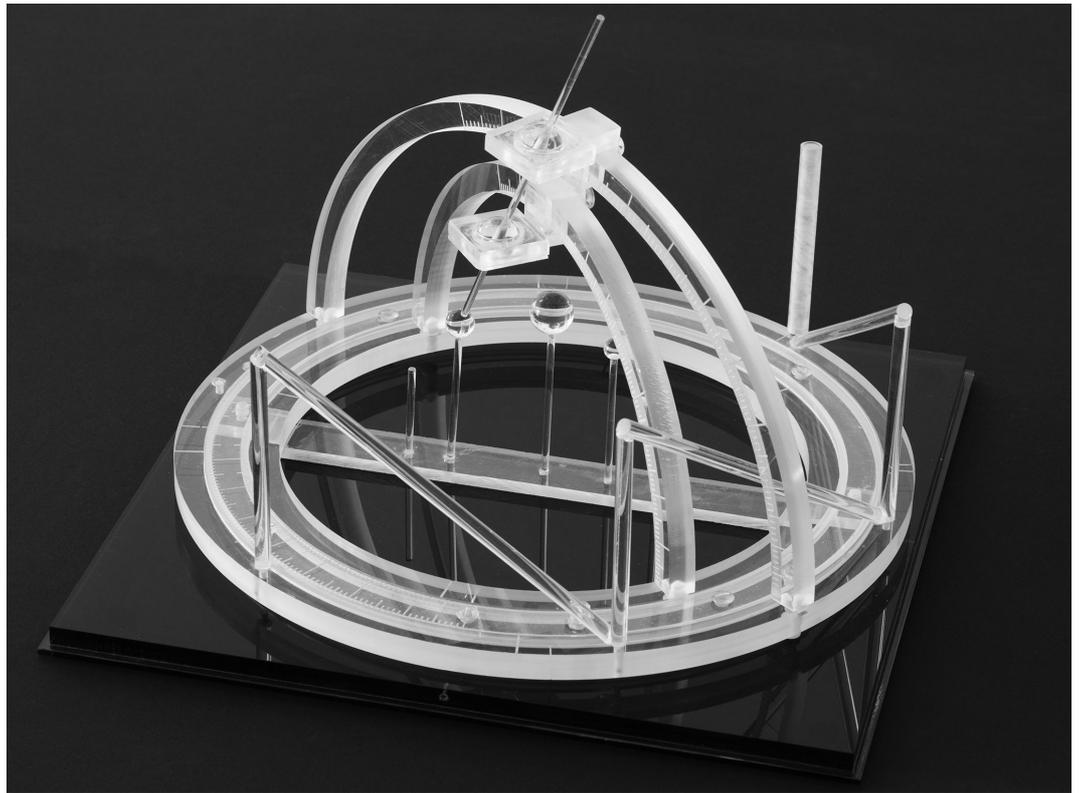
Brown R A, Nelson J A. (2014-01-16 18:37:22 UTC) The History and Mathematics of the N-Localizer for Stereotactic Neurosurgery. Cureus 6(1): e156. DOI 10.7759/cureus.156



**FIGURE 1: N-localizer and its interaction with the scan section**

(a) Side view of the N-localizer. The tomographic section intersects two vertical rods and one diagonal rod. (b) Tomographic image. The intersection of the tomographic section with the N-localizer produces two circles and one ellipse. The relative spacing between the ellipse and the two circles varies according to the height at which the tomographic section intersects the diagonal rod. Measuring this spacing permits calculation of the location of the tomographic section relative to the N-localizer.

Russell A. Brown invented the N-localizer in May 1978 when he was a third-year medical student at the University of Utah and during a research elective in the laboratory of James A. Nelson. In August 1978, Brown built the first CT-compatible stereotactic frame in order to test the concept of the N-localizer (Figure 2). In October 1978, he presented this frame at a joint meeting of the Western Neurological Society and the American Academy of Neurological Surgeons [5]. Following this presentation, additional presentations [6], and disclosures [7], the concept of the N-localizer spread rapidly and inspired its incorporation into three different CT-compatible stereotactic frames that were built in 1979 and reported in 1980: The Brown-Roberts-Wells frame [8], a Leksell frame that was modified to render it CT-compatible [9], and a frame that resulted from a collaboration between Pfizer Medical Systems and the University of Pittsburgh Medical Center (the “UPMC” frame) [10].



**FIGURE 2: The first CT-compatible stereotactic frame**

This stereotactic frame was constructed in 1978 and used to test the concept of the N-localizer by targeting the small spheres inside the frame [3-6].

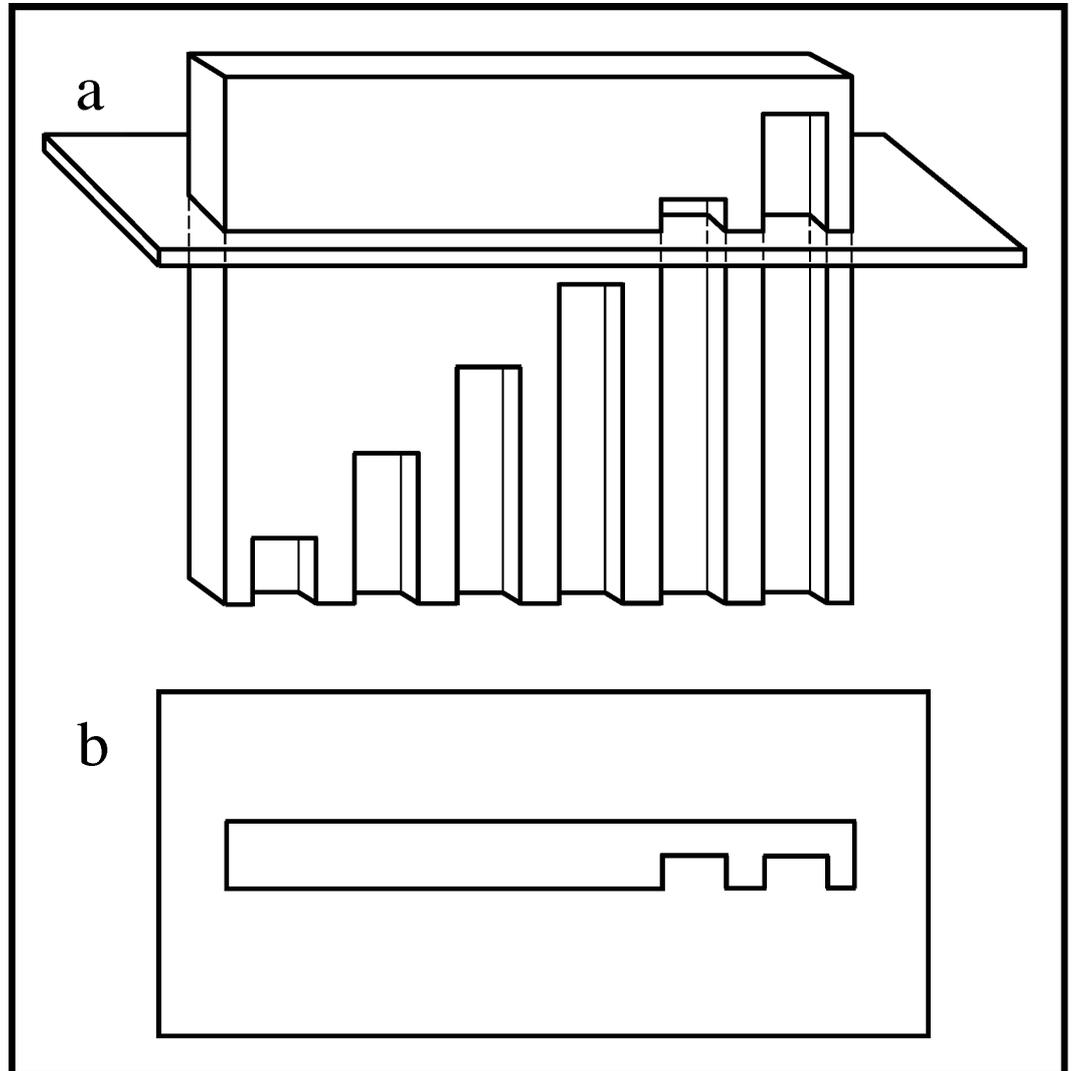
During the 35 years since the invention of the N-localizer, some misconceptions have arisen concerning its history in relation to subsequent developments in image-guided stereotactic surgery. Kondziolka and Lunsford of the University of Pittsburgh School of Medicine have claimed [11], "At our center, the first CT compatible stereotactic head frame, in collaboration with industry, was constructed in 1978 and utilized in 13 patients [10, 12]. [...] During this interval, the newly redesigned Leksell CT compatible stereotactic head frame [9] was used for dedicated brain biopsies under the direction of its inventor, Lars Leksell. Several groups were working on devices to allow accurate CT based stereotactic surgery [13]."

The above statement is not the only instance wherein Lunsford and Kondziolka have inaccurately interpreted the history of image-guided stereotactic surgery; we have discussed another instance previously [1]. But in particular, the above statement presents an erroneous chronology and disregards the fact that the adoption of the N-localizer for the Leksell and UPMC stereotactic frames was derivative. For both of these frames, the knowledge of the N-localizer originated from Brown's prior invention of the N-localizer [1, 9-10]. The UPMC frame was not the first CT-compatible stereotactic frame. And the UPMC frame was built in 1979, not in 1978. The documents that corroborate these facts have remained preserved in the archives of the U.S. Patent and Trademark office for nearly 30 years. The following technical report, which is based on these archives, recounts the development of image-guided stereotactic surgery at the University of Pittsburgh Medical Center and relates the events that led to the construction of the UPMC frame.

## Technical Report

Prior to the invention of the N-localizer, a method had been reported for estimating the location of a tomographic image relative to patient anatomy [14-15]. This method involved a plate into

which were milled vertical slots whose tops lay along a diagonal line (Figure 3). Because this slotted plate required counting numerous notches that were visible in the tomographic image, it was cumbersome to use and susceptible to human error [1, 16]. Despite these limitations, in the summer of 1978, Dade Lunsford modified a Leksell Stalix frame via the addition of two aluminum plates having milled vertical slots in the hope of creating a CT-compatible stereotactic frame [17].



**FIGURE 3: Slotted plate and its interaction with the tomographic section**

(a) Side view of the slotted plate. The tomographic section intersects the plate into which are milled vertical slots. The tops of the slots lie along a diagonal line. (b) Tomographic image. The intersection of the tomographic section with the slotted plate produces a variable number of notches. The number of notches depends on the height at which the tomographic section intersects the plate. Counting the number of notches permits estimation of the location of the tomographic section relative to the slotted plate.

In the autumn of 1978, Lunsford, et al. began to collaborate with John Perry of Pfizer Medical Systems [1, 17]. By mid-January of 1979, Perry, Lunsford, et al. had added a third aluminum plate to the Stalix frame and Perry had developed computer software that used information from the three slotted plates to transform coordinates from a CT scan image into the coordinate system of the Stalix frame [16, 18-19]. In a letter dated January 15, 1979 and addressed to Dade Lunsford, Arthur Rosenbaum and David Zorub of the University of Pittsburgh School of Medicine, Perry provided detailed instructions that explained the procedure required for obtaining CT scan

images of the modified Stalix frame and for using computer software to transform coordinates from a CT scan image into the coordinate system of the Stalix frame [16].

Consequently, in early 1979, Perry, Lunsford, et al. appeared to be at the point of performing clinical cases with the modified Stalix frame. However, according to Lunsford, the modified Stalix frame was never used clinically because the aluminum composition of the frame and attached slotted plates created artifacts in the CT scan image [18]. Lunsford's account is corroborated by Perry, et al. who reported that the UPMC "frame was made after attempts to modify the Leksell frame proved difficult" [10].

Lunsford has asserted that the Stalix frame was abandoned in the fall of 1978 and that thereafter the UPMC frame was constructed [18]. However, he appears to have forgotten the contents of the January 1979 letter from Perry to Lunsford, et al., a copy of which remains preserved in the archives of the U.S. patent office [16]. This letter refutes Lunsford's assertion and clearly demonstrates that in January 1979, Perry, Lunsford, et al. were preparing to use the modified Stalix frame for clinical cases but they had not yet done so. It was only after they had attempted to use the modified Stalix frame that they understood the severity of the artifacts created by the aluminum and subsequently abandoned the Stalix frame [10, 18]. And it was only after Brown had disclosed the N-localizer to Perry in early 1979 that Perry understood the concept of the N-localizer and subsequently adopted the N-localizer for the UPMC frame [1, 7, 10].

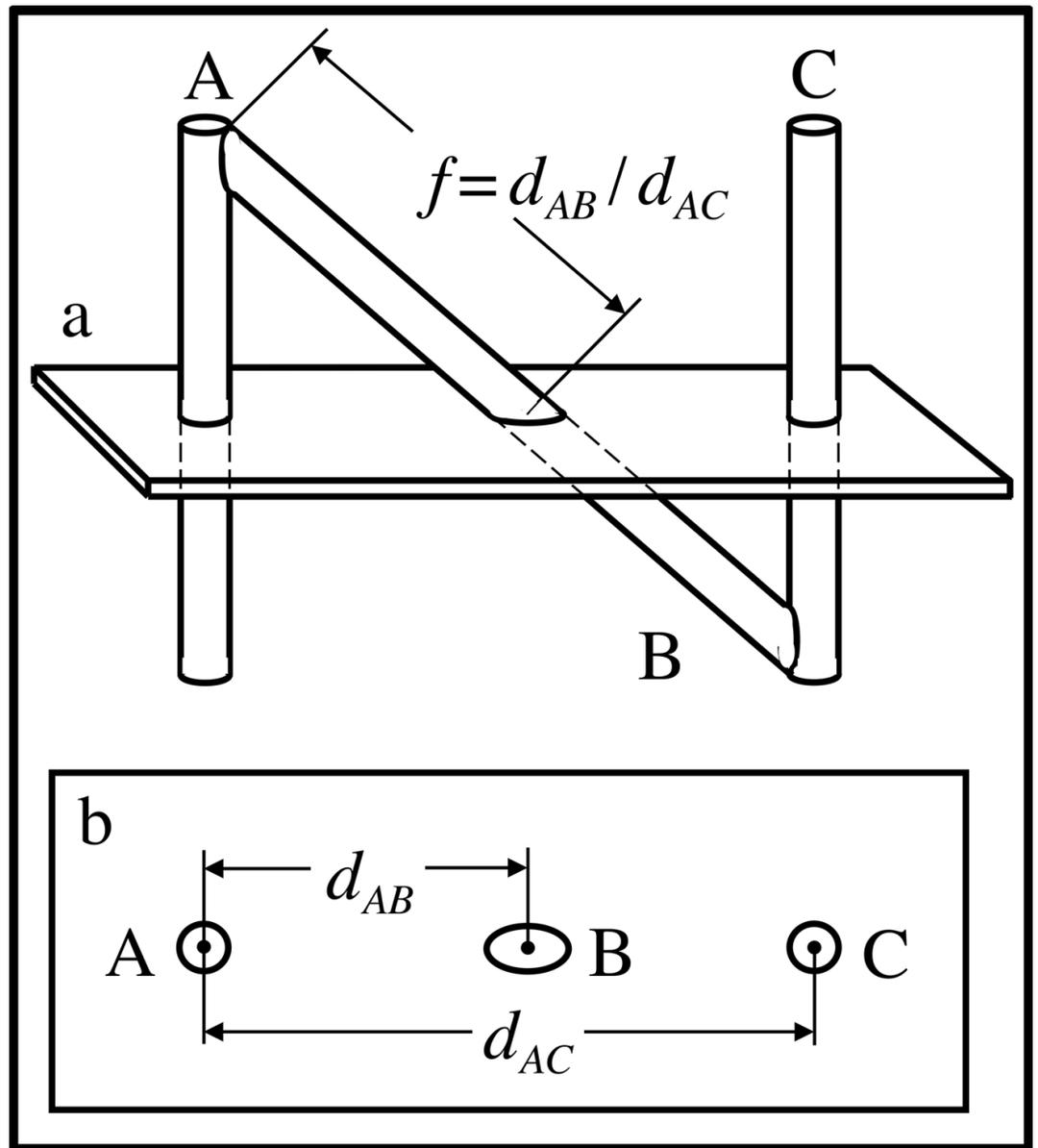
The second [4] of Brown's two publications that announced the N-localizer and reported evaluations of the first CT-compatible stereotactic frame [4-5] was submitted for publication in January 1979. The first [5] of these publications affirms that Brown presented this stereotactic frame at a joint meeting of the Western Neurological Society and the American Academy of Neurological Surgeons in October 1978. These and other publications [4-5, 10], together with historical documents [16, 19] from the archives of the U.S. patent office, confirm that the UPMC frame was not the first CT-compatible stereotactic frame and further confirm that the UPMC frame was constructed in 1979, not in 1978. The relevant historical documents [16, 19] are a matter of public record and may be obtained from the U.S. Patent and Trademark Office by requesting a copy of the folder for Interference No. 101267. In order to facilitate access to these documents, we have included copies as the appendices (labeled as "figures") to this paper.

## Discussion

The mathematics of the N-localizer have been discussed previously [4] and in considerable detail [2]. We review the mathematics below in preparation for presenting further details. The reader who is not interested in the details of the mathematics may choose to skip directly to the Conclusions.

### Review of the mathematics of the N-Localizer

The N-localizer comprises a diagonal rod that extends from the top of one vertical rod to the bottom of another vertical rod (Figure 4). In the tomographic image, each of the two vertical rods creates a fiducial circle and the diagonal rod creates a fiducial ellipse. The ellipse moves away from one circle and towards the other circle as the position of the tomographic section moves upward with respect to the N-localizer. The relative spacing between the three centroids permits precise localization of the tomographic section relative to the N-localizer. The distance  $d_{AB}$  between the centroids of circle A and ellipse B and the distance  $d_{AC}$  between the centroids of circles A and C are used to calculate the ratio  $f = d_{AB}/d_{AC}$ . This ratio represents the fraction of the diagonal rod B that extends from the top of vertical rod A to the point of intersection of rod B with the tomographic section.



**FIGURE 4: Intersection of the tomographic section with the N-localizer**

(a) Side view of the N-localizer. The tomographic section intersects rods A, B and C. (b) Tomographic image. The intersection of the tomographic section with rods A, B and C creates fiducial circles A and C and fiducial ellipse B in the tomographic image. The distance  $d_{AB}$  between the centroids of circle A and ellipse B and the distance  $d_{AC}$  between the centroids of circles A and C are used to calculate the ratio  $f = d_{AB}/d_{AC}$ . This ratio represents the fraction of diagonal rod B that extends from the top of rod A to the point of intersection of rod B with the tomographic section. These geometric relationships are valid, even if the tomographic section is not perpendicular to the vertical rods, as can be demonstrated using similar triangles [5].

The fraction  $f$  is used to calculate the  $(x, y, z)$  coordinates of the point of intersection  $P'_B$  between rod B and the tomographic section (Figure 5). In this figure, points  $P'_A$  and  $P'_C$  represent the beginning and end, respectively, of the vector that extends from the top of rod A to the bottom of rod C. This vector coincides with the long axis of rod B. The  $(x_A, y_A, z_A)$  coordinates of the beginning point  $P'_A$  and the  $(x_C, y_C, z_C)$  coordinates of the end point  $P'_C$  are known from the geometry of the N-localizer. Hence, linear interpolation may be used to

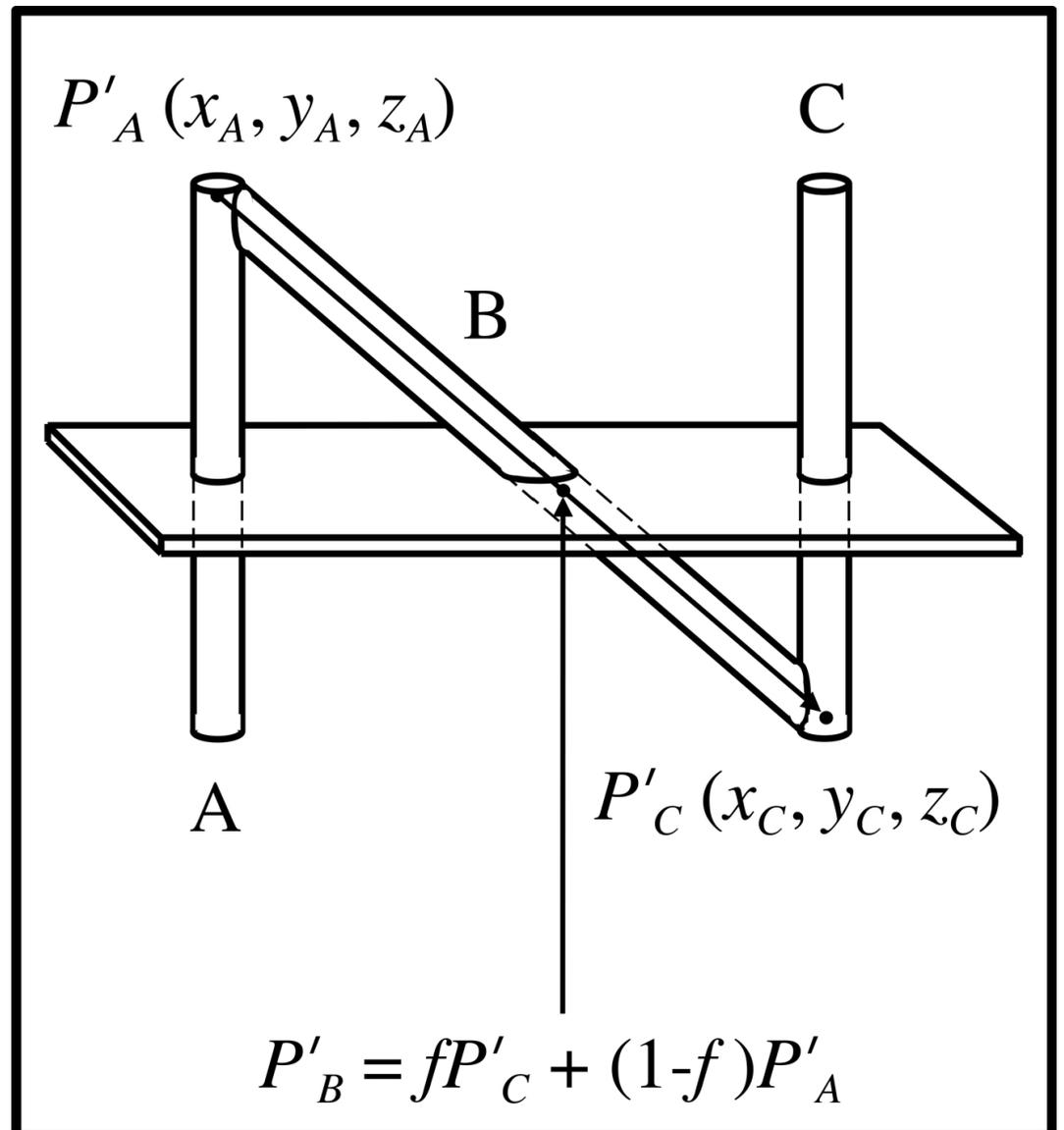
blend points  $P'_A$  and  $P'_C$  to obtain the  $(x_B, y_B, z_B)$  coordinates of the point of intersection  $P'_B$  between the long axis of rod B and the tomographic section, as follows

$$P'_B = P'_A + f(P'_C - P'_A) = fP'_C + (1 - f)P'_A \quad (1)$$

The vector form of Equation 1 shows explicitly the  $(x, y, z)$  coordinates of points  $P'_A$ ,  $P'_B$  and  $P'_C$

$$[x_B \ y_B \ z_B] = f [x_C \ y_C \ z_C] + (1 - f) [x_A \ y_A \ z_A] \quad (2)$$

Equation 1 or 2 may be used to calculate the  $(x_B, y_B, z_B)$  coordinates of the point of intersection  $P'_B$  between rod B and the tomographic section. The point  $P'_B$ , which lies on rod B in the three-dimensional coordinate system of the N-localizer, corresponds to an analogous point  $P_B$ , which lies at the centroid of ellipse B in the two-dimensional coordinate system of the tomographic image. Hence, there is a one-to-one correspondence between a point from the N-localizer and a point from the tomographic image.

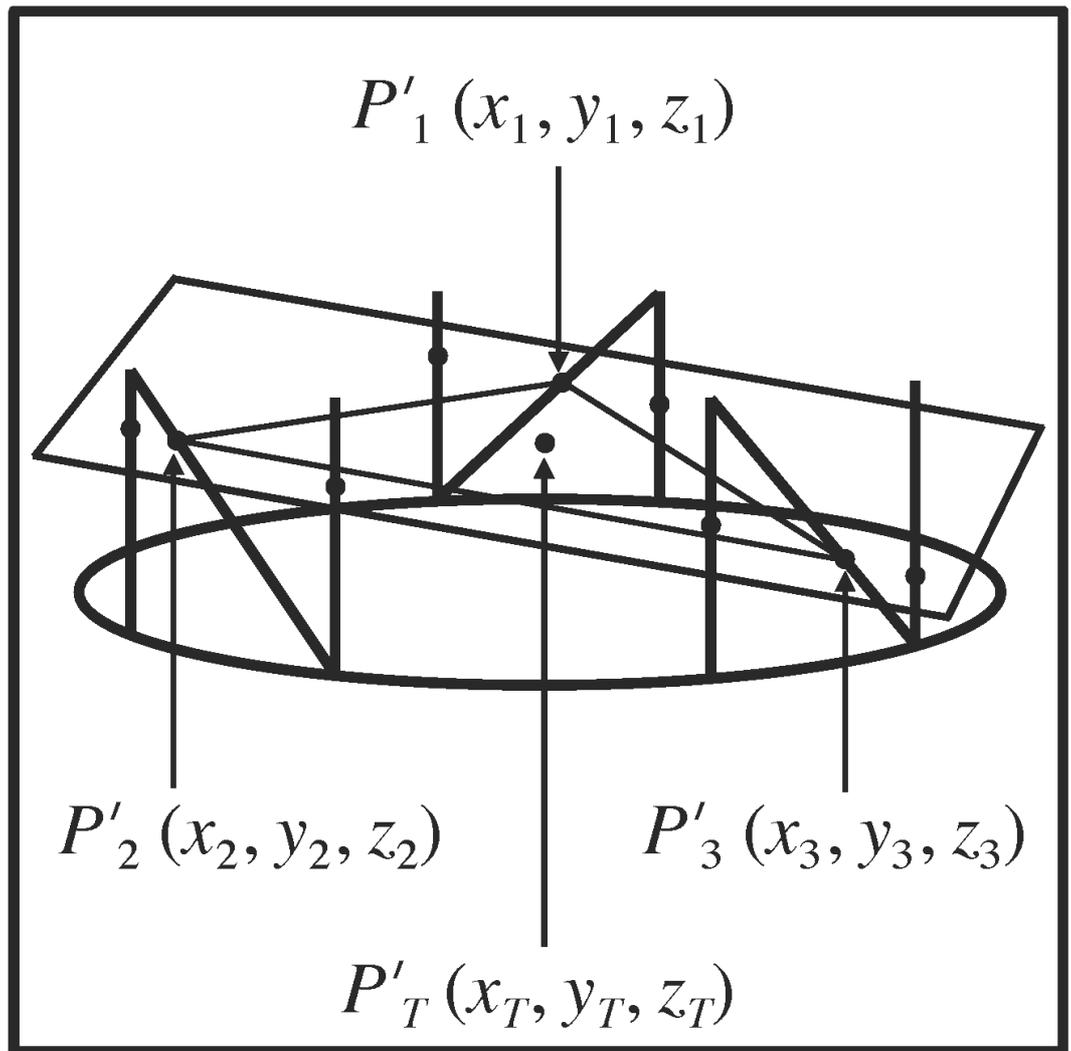


**FIGURE 5:** Calculation of the point of intersection between rod B and the tomographic section  
The long axis of rod B is represented by a vector that extends from point  $P'_A$  at the top of rod A

to point  $P'_C$  at the bottom of rod C. The  $(x_A, y_A, z_A)$  coordinates of point  $P'_A$  and the  $(x_C, y_C, z_C)$  coordinates of point  $P'_C$  are known from the physical dimensions of the N-localizer. Hence, the ratio  $f = d_{AB}/d_{AC}$  may be used to blend the  $(x_A, y_A, z_A)$  and  $(x_C, y_C, z_C)$  coordinates of points  $P'_A$  and  $P'_C$  via linear interpolation as indicated by Equations 1 and 2. This interpolation calculates the  $(x_B, y_B, z_B)$  coordinates of the point of intersection  $P'_B$  between the long axis of rod B and the tomographic section.

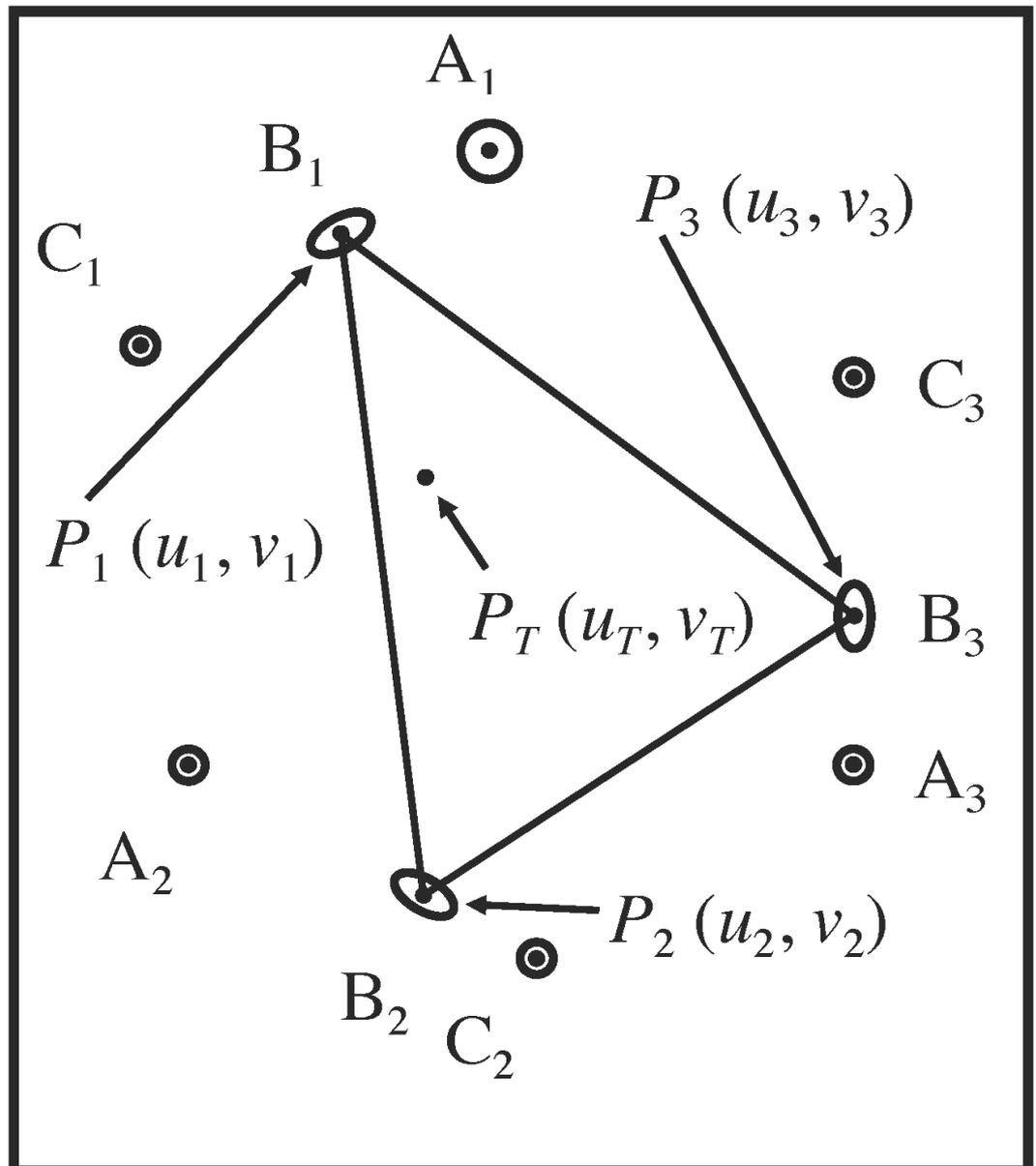
The attachment of three N-localizers to the stereotactic frame permits calculation of the  $(x_{B_1}, y_{B_1}, z_{B_1})$ ,  $(x_{B_2}, y_{B_2}, z_{B_2})$  and  $(x_{B_3}, y_{B_3}, z_{B_3})$  coordinates for the three respective points  $P'_{B_1}$ ,  $P'_{B_2}$  and  $P'_{B_3}$  in the three-dimensional coordinate system of the stereotactic frame. These three points correspond respectively to the three points  $P_{B_1}$ ,  $P_{B_2}$  and  $P_{B_3}$  in the two-dimensional coordinate system of the tomographic image. In the following discussion, the symbols  $P'_1$ ,  $P'_2$  and  $P'_3$  will be used as a shorthand notation for  $P'_{B_1}$ ,  $P'_{B_2}$  and  $P'_{B_3}$ . The symbols  $P_1$ ,  $P_2$  and  $P_3$  will be used as a shorthand notation for  $P_{B_1}$ ,  $P_{B_2}$  and  $P_{B_3}$ .

The three points  $P'_1$ ,  $P'_2$  and  $P'_3$  lie on the three respective diagonal rods  $B_1$ ,  $B_2$  and  $B_3$  and have respective  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates in the three-dimensional coordinate system of the stereotactic frame (Figure 6). The analogous three points  $P_1$ ,  $P_2$  and  $P_3$  lie at the centroids of the three respective ellipses  $B_1$ ,  $B_2$  and  $B_3$  and have  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates in the two-dimensional coordinate system of the tomographic image (Figure 7). Because three points determine a plane in three-dimensional space, the three coordinates  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  together with the analogous three coordinates  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  determine the spatial orientation of the tomographic section relative to the stereotactic frame. This spatial orientation permits calculation of the  $(x_T, y_T, z_T)$  coordinates of the target point  $P'_T$  in the three-dimensional coordinate system of the stereotactic frame, given the  $(u_T, v_T)$  coordinates of the analogous target point  $P_T$  in the two-dimensional coordinate system of the tomographic image.



**FIGURE 6: Representation of the tomographic section in the three-dimensional coordinate system of the stereotactic frame**

The quadrilateral represents the tomographic section. The large oval depicts the base of the stereotactic frame. The vertical and diagonal lines that are attached to the large oval represent the nine rods. The centroids of the six fiducial circles and the three fiducial ellipses that are created in the tomographic image by these nine rods are shown as points that lie in the tomographic section. The tomographic section intersects the three diagonal rods at points  $P'_1$ ,  $P'_2$  and  $P'_3$  that coincide with the respective centroids  $P_1$ ,  $P_2$  and  $P_3$  of the three ellipses (Figure 7). The  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates of the respective points of intersection  $P'_1$ ,  $P'_2$  and  $P'_3$  are calculated in the three-dimensional coordinate system of the stereotactic frame using Equations 1 and 2. Because these three points determine a plane in three-dimensional space, the spatial orientation of the tomographic section is determined relative to the stereotactic frame. The target point  $P'_T$  lies in the tomographic section. The  $(x_T, y_T, z_T)$  coordinates of this target point are calculated in the three-dimensional coordinate system of the stereotactic frame using Equation 14. The target point  $P'_T$  lies in the plane of the triangle  $P'_1P'_2P'_3$  and hence its  $(x_T, y_T, z_T)$  coordinates may be expressed as a linear combination of the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates of points  $P'_1$ ,  $P'_2$  and  $P'_3$  using barycentric coordinates via Equation 7[20].



**FIGURE 7: Representation of the two-dimensional coordinate system of the tomographic image**

Three N-localizers create three sets of fiducial marks  $\{A_1, B_1, C_1\}$ ,  $\{A_2, B_2, C_2\}$ , and  $\{A_3, B_3, C_3\}$  in the tomographic image. Each set contains two circles and one ellipse. The centroids  $P_1, P_2$  and  $P_3$  of the three ellipses coincide with the respective points of intersection  $P'_1, P'_2$  and  $P'_3$  of the three diagonal rods with the tomographic section (Figure 6). The  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates of the centroids  $P_1, P_2$  and  $P_3$  correspond respectively to the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates of the points of intersection  $P'_1, P'_2$  and  $P'_3$ . A target point  $P_T$  has  $(u_T, v_T)$  coordinates in the two-dimensional coordinate system of the tomographic image. The  $(x_T, y_T, z_T)$  coordinates of the analogous target point  $P'_T$  are calculated in the three-dimensional coordinate system of the stereotactic frame using Equation 14. The target point  $P_T$  lies in the plane of the triangle  $P_1P_2P_3$  and hence its  $(u_T, v_T)$  coordinates may be expressed as a linear combination of the  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates of points  $P_1, P_2$  and  $P_3$  using barycentric coordinates via Equation 8 [20].

In order to facilitate calculation of the  $(x_T, y_T, z_T)$  coordinates of the target point  $P'_T$ , it is

convenient to project the  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates of the three respective points  $P_1$ ,  $P_2$  and  $P_3$  onto the  $w = 1$  plane in three-dimensional space by appending a third coordinate  $w = 1$  to create  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates. The  $w$ -coordinate may be set arbitrarily to any non-zero value, e.g., one, so long as same value of  $w$  is used for each of the three  $w$ -coordinates. The equations that are presented in the remainder of this paper assume that a value of  $w = 1$  has been used to project the  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates. If a value of  $w \neq 1$  were used instead of  $w = 1$  to project the  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates, the equations that are presented in the remainder of this paper would no longer apply and would require revision so that the calculations that these equations describe may produce correct results.

The correspondence, or transformation, between the two-dimensional coordinate system of the tomographic image and the three-dimensional coordinate system of the stereotactic frame may be represented using the three  $(x, y, z)$  and  $(u, v, 1)$  coordinate pairs  $(x_1, y_1, z_1)$  and  $(u_1, v_1, 1)$ ;  $(x_2, y_2, z_2)$  and  $(u_2, v_2, 1)$ ; and  $(x_3, y_3, z_3)$  and  $(u_3, v_3, 1)$  in the following matrix equation

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad (3)$$

Equation 3 represents concisely a system of nine simultaneous linear equations that determine the spatial orientation of the tomographic section relative to the stereotactic frame. This equation transforms the  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates from the two-dimensional coordinate system of the tomographic image to create  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates in the three-dimensional coordinate system of the stereotactic frame.

An analogy provides insight into how the transformation of Equation 3 operates. Consider the tomographic image to be an elastic membrane. The transformation describes the processes of stretching the membrane in the plane of the tomographic image, rotating the membrane about an axis that is normal to the plane of the tomographic image, tilting the membrane if necessary so that it is not parallel to the base of the stereotactic frame, and lastly, lifting the membrane into place upon the scaffold of the three N-localizers such that the three points  $P_1$ ,  $P_2$  and  $P_3$  from the tomographic image precisely coincide with the respective three points  $P'_1$ ,  $P'_2$  and  $P'_3$  from the stereotactic frame. Then *any* other point that lies on the membrane, for example, the target point  $P_T$ , is transformed by the same stretching, rotating, tilting and lifting processes that transformed the three points  $P_1$ ,  $P_2$  and  $P_3$ . In this manner, the  $(u_T, v_T)$  coordinates of the target point  $P_T$  may be transformed from the two-dimensional coordinate system of the tomographic image into the three-dimensional coordinate system of the stereotactic frame to produce the  $(x_T, y_T, z_T)$  coordinates of the analogous target point  $P'_T$ .

Equation 3 may be written in more compact form as

$$\mathbf{F} = \mathbf{S}\mathbf{M} \quad (4)$$

In Equation 4,  $\mathbf{F}$  represents the matrix of  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates from the coordinate system of the stereotactic frame.  $\mathbf{S}$  represents the matrix of  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates from the coordinate system of the tomographic image.  $\mathbf{M}$  represents the matrix of elements  $m_{11}$  through  $m_{33}$  that defines the transformation from the two-dimensional coordinate system of the tomographic image into the three-dimensional coordinate system of the stereotactic frame.

Solving Equation 4 for  $\mathbf{S}$  yields

$$\mathbf{S} = \mathbf{F}\mathbf{M}^{-1} \quad (5)$$

Equation 5 transforms the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates from the three-dimensional coordinate system of the stereotactic frame to create  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates in the two-dimensional coordinate system of the tomographic image.

The above review of the mathematics of the N-localizer is sufficient to permit the presentation of further details below.

## Requirements for the matrices $\mathbf{F}$ and $\mathbf{S}$

It is possible to solve Equation 4 for the transformation matrix  and this solution implies that  $\mathbf{M}$  exists so long as the inverse matrix  $\mathbf{S}^{-1}$  exists. It is possible to solve Equation 5 for the inverse transformation matrix  $\mathbf{M}^{-1} = \mathbf{F}^{-1}\mathbf{S}$  and this solution implies that  $\mathbf{M}^{-1}$  exists so long as the inverse matrix  $\mathbf{F}^{-1}$  exists.

From the above considerations, it is apparent that Equations 4 and 5 require that the inverse matrices  $\mathbf{S}^{-1}$  and  $\mathbf{F}^{-1}$  exist.  $\mathbf{S}^{-1}$  exists as long as the points  $P_1$ ,  $P_2$  and  $P_3$  are not collinear and no column of  $\mathbf{S}$  contains three zeros.  $\mathbf{F}^{-1}$  exists as long as the points  $P'_1$ ,  $P'_2$  and  $P'_3$  are not collinear and no column of  $\mathbf{F}$  contains three zeros.

The collinearity requirement is satisfied by the positions of the three N-localizers relative to the stereotactic frame. Because for contemporary stereotactic frames the three N-localizers are positioned either around the circumference of a circle or on the faces of a cube, neither the points  $P_1$ ,  $P_2$  and  $P_3$  nor the points  $P'_1$ ,  $P'_2$  and  $P'_3$  can possibly be collinear.

The requirement that no column of  $\mathbf{S}$  contain three zeros is satisfied by choosing  $w = 1$  and not  $w = 0$  when projecting the  $(u_1, v_1)$ ,  $(u_2, v_2)$  and  $(u_3, v_3)$  coordinates to create  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates.

The requirement that no column of  $\mathbf{F}$  contain three zeros may be satisfied by defining the three-dimensional coordinate system of the stereotactic frame such that the  $z$ -coordinate does not equal zero anywhere along the diagonal rods. Figure 5 demonstrates that Equations 1 and 2 will never produce  $z = 0$  along the diagonal rods so long as the interval from  $z_A$  to  $z_C$  does not contain zero. One way to satisfy this requirement is to define the origin of the three-dimensional coordinate system of the stereotactic frame to be the corner of a cube that surrounds the stereotactic frame, such that the  $z$ -coordinate of the origin is less than the  $z_C$ -coordinate of point  $P_C$ .

When all three of the above requirements are satisfied, the matrix  $\mathbf{M}$  will correctly transform the  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates of points  $P_1$ ,  $P_2$  and  $P_3$  from the two-dimensional coordinate system of the tomographic image to create the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates of points  $P'_1$ ,  $P'_2$  and  $P'_3$  in the three-dimensional coordinate system of the stereotactic frame. And the inverse matrix  $\mathbf{M}^{-1}$  will correctly perform the inverse transformation.

## Transformation of the target point $P_T$ and the analogous target point $P'_T$

The above analysis addresses the transformation of points  $P_1$ ,  $P_2$  and  $P_3$  and analogous

points  $P'_1$ ,  $P'_2$  and  $P'_3$ . However, the above discussion of the membrane analogy claims that *any* point that lies in the plane of points  $P_1$ ,  $P_2$  and  $P_3$  may be transformed from the two-dimensional coordinate system of the tomographic section into the three-dimensional coordinate system of the stereotactic frame via the same transformation that transforms points  $P_1$ ,  $P_2$  and  $P_3$ . The following analysis proves this assertion.

Equation 3 transforms en masse the  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates of the three points  $P_1$ ,  $P_2$  and  $P_3$  to create the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates of the three points  $P'_1$ ,  $P'_2$  and  $P'_3$ . An alternative is to transform separately the  $(u_1, v_1, 1)$ ,  $(u_2, v_2, 1)$  and  $(u_3, v_3, 1)$  coordinates of the three points  $P_1$ ,  $P_2$  and  $P_3$

$$P'_1 = [x_1 \ y_1 \ z_1] = [u_1 \ v_1 \ 1] \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = P_1 \mathbf{M} \quad (6a)$$

$$P'_2 = [x_2 \ y_2 \ z_2] = [u_2 \ v_2 \ 1] \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = P_2 \mathbf{M} \quad (6b)$$

$$P'_3 = [x_3 \ y_3 \ z_3] = [u_3 \ v_3 \ 1] \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = P_3 \mathbf{M} \quad (6c)$$

Equation 6 produces the same result for the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  coordinates of points  $P'_1$ ,  $P'_2$  and  $P'_3$  as does Equation 3.

It is possible to represent any point that lies in the plane defined by three other points as a linear combination of those three points. For example, with reference to Figure 6, the target point  $P'_T$  may be represented as a linear combination of the three points  $P'_1$ ,  $P'_2$  and  $P'_3$

$$P'_T = b_1 P'_1 + b_2 P'_2 + b_3 P'_3 \quad (7)$$

Similarly, with reference to Figure 7, the analogous target point  $P_T$  may be represented as a linear combination of the three points  $P_1$ ,  $P_2$  and  $P_3$ .

$$P_T = b_1 P_1 + b_2 P_2 + b_3 P_3 \quad (8)$$

As used in Equations 7 and 8, the terms  $b_1, b_2$  and  $b_3$  are known as *barycentric* coordinates and satisfy the condition  $b_1 + b_2 + b_3 = 1$  [20]. Because the matrix  $\mathbf{M}$  that transforms  $\mathbf{S}$  into  $\mathbf{F}$  describes a linear transformation, the same barycentric coordinates  $b_1, b_2$  and  $b_3$  apply to either  $P_T$  or  $P'_T$  and hence may be used in both Equation 7 and Equation 8. These equations describe interpolation in a plane that is analogous to the interpolation along a line that is expressed by Equation 1.

Using matrix  $\mathbf{M}$  to transform point  $P_T$  as suggested by Equation 4 and substituting Equations 6-8 yields

$$\begin{aligned} P_T \mathbf{M} &= (b_1 P_1 + b_2 P_2 + b_3 P_3) \mathbf{M} \\ &= b_1 P_1 \mathbf{M} + b_2 P_2 \mathbf{M} + b_3 P_3 \mathbf{M} \\ &= b_1 P'_1 + b_2 P'_2 + b_3 P'_3 \\ &= P'_T \end{aligned} \tag{9}$$

Eliminating the intermediate steps from Equation 9 and showing explicitly the  $(x_T, y_T, z_T)$  coordinates of  $P'_T$  and the  $(u_T, v_T)$  coordinates of  $P_T$  yields

$$P'_T = [x_T \ y_T \ z_T] = [u_T \ v_T \ 1] \mathbf{M} = P_T \mathbf{M} \tag{10}$$

Equation 10 proves that any point  $P_T$  that lies in the tomographic image may be transformed from the two-dimensional coordinate system of that image into the three-dimensional coordinate system of the stereotactic frame to obtain the analogous point  $P'_T$ .

Using matrix  $\mathbf{M}^{-1}$  to transform point  $P'_T$  as suggested by Equation 5 and substituting Equations 6-8 yields

$$\begin{aligned} P'_T \mathbf{M}^{-1} &= (b_1 P'_1 + b_2 P'_2 + b_3 P'_3) \mathbf{M}^{-1} \\ &= b_1 P'_1 \mathbf{M}^{-1} + b_2 P'_2 \mathbf{M}^{-1} + b_3 P'_3 \mathbf{M}^{-1} \\ &= b_1 P_1 + b_2 P_2 + b_3 P_3 \\ &= P_T \end{aligned} \tag{11}$$

Eliminating the intermediate steps from Equation 11 and showing explicitly the  $(x_T, y_T, z_T)$  coordinates of  $P'_T$  and the  $(u_T, v_T)$  coordinates of  $P_T$  yields

$$P_T = [u_T \ v_T \ 1] = [x_T \ y_T \ z_T] \mathbf{M}^{-1} = P'_T \mathbf{M}^{-1} \tag{12}$$

Equation 12 proves that any point  $P'_T$  that lies in the plane of the tomographic section may be transformed from the three-dimensional coordinate system of the stereotactic frame into the

two-dimensional coordinate system of the tomographic image to obtain the analogous point  $P_T$ .

Solving Equation 4 for  $\mathbf{M}$  yields

$$\mathbf{M} = \mathbf{S}^{-1}\mathbf{F} \quad (13)$$

Substituting Equation 13 into Equation 10 and showing explicitly the elements of matrices  $\mathbf{S}$  and  $\mathbf{F}$  from Equation 3 yields the transformation equation that accompanied the introduction of the Brown-Roberts-Wells stereotactic frame [8]

$$\begin{bmatrix} x_T & y_T & z_T \end{bmatrix} = \begin{bmatrix} u_T & v_T & 1 \end{bmatrix} \begin{bmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \quad (14)$$

## Conclusions

The first CT-compatible stereotactic frame, and the only CT-compatible stereotactic frame that was constructed in 1978, is the frame that Brown built in order to test the concept of the N-localizer. Dissemination of the concept of the N-localizer inspired its incorporation into three different CT-compatible stereotactic frames that were constructed in 1979 and reported in 1980: the Brown-Roberts-Wells frame, a Leksell frame that was modified to render it CT-compatible, and the UPMC frame. The Brown-Roberts-Wells frame, its successor the Cosman-Roberts-Wells frame, and the Leksell frame all achieved widespread clinical use. The UPMC frame was utilized in 13 patients and then abandoned.

The N-localizer permits determination of the spatial orientation of a tomographic section relative to the stereotactic frame. When used to the full extent of its capability, the N-localizer allows calculation of the spatial orientation of a tomographic section that is arbitrarily oriented relative to the frame. This method obviates the need to affix the frame to the tomographic scanner bed or to align the frame to the scanner [3-6, 8, 13, 21]. It is possible to forgo these advantages and instead employ the N-localizer in a restricted manner that requires fixation of the frame to the scanner bed and also requires careful alignment of the frame to ensure that the tomographic section is parallel to the base of the frame [9].

The impetus for this restricted use of the N-localizer appears to have been concern over the N-localizer's requirement for "advanced computer technology" [9]. However, 30 to 35 years ago, exploiting the full capability of the N-localizer required only the CT scanner computer [4-6, 8, 13] or even a mere hand-held, programmable calculator [13, 21] to perform the calculation indicated by Equation 14. At that time, neither the CT scanner computer nor the hand-held calculator exemplified advanced computer technology. In this era of modern neurosurgery, the computation required to plan stereotactic radiosurgery far exceeds the minuscule computation required to harness the full capability of the N-localizer.

## Appendices



January 15, 1979

Dr. Dede Lunsford  
 Dr. Arthur Rosenbaum  
 Dr. David Strub  
 University of Pittsburgh  
 School of Medicine  
 Pittsburgh, Pennsylvania 15261

Gentlemen:  
 I hope in our brief time together on Sunday you felt we accomplished your goals. Here is my description of the technique.  
 Once the fiducial plate file is set up for the frame, I think the clinical sequence of events will go something like this:

1. Mount the frame to the patient's head.
2. Attach the fiducial plates to the frame.
3. Scan the patient at the narrowest slice thickness until the target is found in the scan (this must be accomplished in a slice which intersects each fiducial plate at least one slot from each end, eg, more than one and less than the total number of slots must be seen in each of the three fiducial plates).
4. Fill out the Stereotaxis Data Sheet:  
 Note the image scale factors.  
 For each fiducial:  
 a. Count the number of slots seen.  
 b. Using the cursor, measure the CT scan coordinates of the slot which ends in the plane of the slice.  
 Then similarly measure the CT scan coordinates of the target.
5. Run the 'AIII' program, typing in the requested data and recording the calculated frame coordinates of the target on the data sheet.
6. Check the comparisons of the lengths of vectors A, B, and C. We know that 1 m accuracy can be obtained with differences of up to 0.35 cm. A large difference indicates something is wrong.
7. Remove the fiducial plates.
8. Carry out the classical stereotaxis procedure to be assured that the computer-calculated coordinates are consistent with the anatomy.
9. Set the calculated Z coordinate into the probe carrier.
10. Set the side bars at the calculated Y coordinate.
11. Attach the probe carrier to the side bars at the calculated X coordinate. At this point the target is exactly centered at the origin of the frame's spherical coordinate system. Any angle (theta or phi) may be selected for the approach.

802 Old Annapolis Road Columbia, Maryland 21045

EXHIBIT B

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12. Check that the depth indicator on the probe carrier is set at zero.
13. Check that the probe length stop is correct using the length fixture.
14. Insert the probe.

In time, I hope to eliminate the transcribing of the coordinates and the rekeying of them into the 'AIII' program.  
 Two critically important points should be made about the procedure. First, the CT scan resolution element is large enough to cause errors of more than 1 mm, therefore, great care must be taken in reading CT scan coordinates and in counting fiducial bars. There is very little the computer can do to check the accuracy of its input. Second, the slice thickness is even the best circumstances will substantially contribute to the error unless care is taken to center the target in the plane of the slice. I think one of the more subtle contributions of the radiologist in this procedure will be his use of the partial volume effect to assure that the target is centered in the slice thickness, minimizing this potentially significant error. In this same vein, I would suggest that in any procedure, care be taken to aim for the center of the target volume.  
 The mathematics of our technique can be described as follows. The basic problem is to measure the coordinate transform between the coordinate system of the stereotaxis frame and the CT scanner. With this transform the CT coordinates of a target can be converted to the corresponding frame coordinates.  
 To me, the innovation in our method is the technique for getting all the information on the transform and the target from one scan, thus minimizing the errors. The transform information is provided by three fiducial plates. With three non-collinear points measured in both coordinate systems, the transform can be calculated. The plates have a sequence of parallel slots of varying length such that by counting the slots which appear in the CT image one knows which slot ends in the plane of the slice. The plates are designed so that the frame coordinates of the end of each slot are known. Thus, counting the number of slots provides the frame coordinates of the fiducial. The CT scan coordinates of the fiducials are measured from the CT image.  
 There are several ways to actually calculate the transform. I approach it as a physicist might.

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Let capital letters denote frame coordinates and lower case letters denote CT scan coordinates, and let:

$$\begin{aligned} F_1, F_2 &= \text{fiducial 1} \\ F_2, F_3 &= \text{fiducial 2} \\ F_1, F_3 &= \text{fiducial 3} \\ T, T &= \text{target} \end{aligned}$$

Define:

$$\begin{aligned} \tilde{A} &= F_2 - F_1 & \tilde{B} &= F_3 - F_1 \\ \tilde{B} &= F_3 - F_1 & \tilde{C} &= T - F_1 \\ \tilde{A} &= T - F_1 & \tilde{C} &= T - F_1 \end{aligned}$$

The problem is to find  $T$ , the target frame coordinates.  
 Since  $\tilde{A}$ ,  $\tilde{B}$ , and  $\tilde{C}$  lie in the same plane and  $\tilde{A}$  and  $\tilde{B}$  are not collinear, we can find by solution of the simultaneous equations,  $a$  and  $b$  such that:

$$\tilde{C} = a\tilde{A} + b\tilde{B}$$

The equivalent in frame coordinates is:

$$\tilde{C} = T - F_1 = a\tilde{A} + b\tilde{B}$$

Thus:

$$T = F_1 + a\tilde{A} + b\tilde{B}$$

As you can see, the problem is trivial, mathematically. If it turns out to be medically useful, I think this will be another strong statement on behalf of CI. I look forward to visiting Pittsburgh in the very near future.

Sincerely,  
 John H. Perry  
 Director  
 Research and Development

JHP/cw

FIGURE 8: Appendix 1: John Perry Letter, pp. 1-3, January 15, 1979



## Additional Information

### Disclosures

**Conflicts of interest:** The authors have declared that no conflicts of interest exist.

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